

# Lösungen Algebra-Test, 26.3.09

① a)  $\frac{1}{\sqrt{27}} = \frac{1}{\sqrt{27}} \cdot \frac{\sqrt{27}}{\sqrt{27}} = \frac{\sqrt{27}}{27} = \frac{\sqrt{3 \cdot 3 \cdot 3}}{27} = \frac{3\sqrt{3}}{27} = \frac{\sqrt{3}}{9}$

b)  $\frac{4}{\sqrt{7}-\sqrt{3}} = \frac{4}{\sqrt{7}-\sqrt{3}} \cdot \frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}+\sqrt{3}} = \frac{4(\sqrt{7}+\sqrt{3})}{7-3} = \frac{4(\sqrt{7}+\sqrt{3})}{4} = \underline{\underline{\sqrt{7}+\sqrt{3}}}$

② a)  $\sqrt{2} \cdot \sqrt{8} = \sqrt{2 \cdot 2 \cdot 2 \cdot 2} = \underline{\underline{4}}$

b)  $\sqrt[3]{10} \cdot \sqrt[3]{100} = \sqrt[3]{10 \cdot 10 \cdot 10} = \underline{\underline{10}}$

③  $\sqrt{5} \cdot \sqrt[4]{5} = 5^{\frac{1}{2}} \cdot 5^{\frac{1}{4}} = 5^{\frac{1}{2} + \frac{1}{4}} = 5^{\frac{3}{4}} = \underline{\underline{\sqrt[4]{5^3}}}$

④  $(x-2)^4 = x^4 + 4x^3(-2)^1 + 6x^2(-2)^2 + 4x(-2)^3 + (-2)^4$   
 $= \underline{\underline{x^4 - 8x^3 + 24x^2 - 32x + 16}}$

		1		1			
		1	2	1			
	1	3	3	1			
1	4	6	4	1			

⑤  $\frac{y-x}{5x-5y} = \frac{-1(x-y)}{5(x-y)} = \underline{\underline{-\frac{1}{5}}}$

⑥  $\frac{\frac{7a-7b}{a+b}}{\frac{a^2-b^2}{a^2+2ab+b^2}} = \frac{\frac{7(a-b)}{a+b}}{\frac{(a+b)(a-b)}{(a+b)^2}} = \frac{7(a-b)}{(a+b)} \cdot \frac{(a+b)^2}{(a+b)(a-b)} = \underline{\underline{7}}$

⑦  $(x^4 + 4x^3 + 4x^2 + 4x + 3) : (x+3) = \underline{\underline{x^3 + x^2 + x + 1}}$

$$\begin{array}{r} x^4 + 3x^3 \\ \hline x^3 + 4x^2 + 4x + 3 \\ x^3 + 3x^2 \\ \hline x^2 + 4x + 3 \\ x^2 + 3x \\ \hline x + 3 \\ x + 3 \\ \hline 0 \end{array}$$

$$\textcircled{8} \quad \frac{1+x}{x^2-x^3} - \frac{1}{x^2} = \frac{1+x}{x^2(1-x)} - \frac{1}{x^2} = \frac{1+x}{x^2(1-x)} - \frac{(1-x)}{x^2(1-x)}$$

$$= \frac{1+x-(1-x)}{x^2(1-x)} = \frac{1+x-1+x}{x^2(1-x)} = \frac{2x}{x^2(1-x)} = \underline{\underline{\frac{2}{x(1-x)}}}$$

$$\textcircled{9} \quad \frac{x}{x^2-1} = \frac{x}{x-1} - \frac{x+2}{x+1} \quad \frac{2}{3} = 2 - \frac{4}{3} \quad \mathbb{D} = \mathbb{R} \setminus \{\pm 1\}$$

$$\Leftrightarrow \frac{x}{(x+1)(x-1)} = \frac{x}{x-1} + \frac{x+2}{x+1} \quad / \cdot (x+1)(x-1)$$

$$x = x(x+1) + (x+2)(x-1)$$

$$x = x^2 + x + x^2 + x + 2 \Rightarrow \underline{\underline{x = 2}} \quad \mathbb{L} = \{2\}$$

$$\textcircled{10} \quad \frac{5}{x-1} + \frac{x-6}{x^2-x} = \frac{5}{x}$$

$$\frac{5}{x-1} + \frac{x-6}{x(x-1)} = \frac{5}{x} \quad / \cdot x(x-1) \quad \mathbb{D} = \mathbb{R} \setminus \{0, 1\}$$

$$5x + (x-6) = 5(x-1)$$

$$5x + x - 6 = 5x - 5$$

$$\underline{\underline{x = 1}}$$

$$\mathbb{L} = \{ \} \quad 1 \notin \mathbb{D}$$

$$\textcircled{11} \quad \sqrt{x+1} = \sqrt{x+8} - 1 \quad / \cdot ^2 \quad \mathbb{D} = \{x \in \mathbb{R} / x \geq -8\}$$

$$(\sqrt{x+1})^2 = (\sqrt{x+8} - 1)^2$$

$$x+1 = (\sqrt{x+8})^2 - 2\sqrt{x+8} + 1$$

$$x+1 = x+8 - 2\sqrt{x+8} + 1 \quad / + 2\sqrt{x+8}$$

$$2\sqrt{x+8} = 8$$

$$\sqrt{x+8} = 4 \quad / \cdot ^2$$

$$x+8 = 16$$

$$\underline{\underline{x = 8}}$$

$$\mathbb{L} = \{8\}$$

$$\textcircled{12} \text{ a) } 10100 = 1 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 0 \cdot 2^0 = 20$$

$$\text{b) } 21 = 16 + 4 + 1 = \underline{1} \cdot 2^4 + \underline{0} \cdot 2^3 + \underline{1} \cdot 2^2 + \underline{0} \cdot 2^1 + \underline{1} \cdot 2^0$$

$$= \underline{\underline{10101_2}}$$